The long term stability of EAL and TAI (revisited)

G. Petit
Bureau International des Poids et Mesures (BIPM)
gpetit@bipm.org

Abstract**—***International Atomic Time TAI gets its stability from some 300 atomic clocks worldwide that generate the free atomic scale EAL and its accuracy from a small number of primary frequency standards (PFS) which frequency measurements are used to steer the EAL frequency. While the long-term (above one year) stability of TAI is mostly driven by the PFS and therefore directly related to its accuracy, its stability for averaging times from one to a few months is mostly driven by EAL. Based on the analysis of 8 years (1999-2006) of TAI data, the paper presents several results. We estimate the long-term stability of EAL and TAI with respect to the latest realization of TT(BIPM), and the one-month stability of EAL from the statistics of the weighted clocks. We study the two principal classes of clocks which form EAL and evidence systematic rate variations in clock ensembles. Their effect on the long-term stability of EAL is discussed.*

I. INTRODUCTION

International Atomic Time TAI gets its stability from some 300 atomic clocks kept in more than 50 laboratories worldwide and its accuracy from a small number of primary frequency standards (PFS) developed by a few metrology laboratories. To be more specific, in the computation of TAI, a free-running time scale, EAL, is first established from a weighted average of some 300 atomic clocks, then the frequency of EAL is compared with that of the PFSs using the algorithm presented in [1]. Then a frequency shift (frequency steering correction) is applied to EAL to ensure that the frequency of TAI conforms to its definition. Changes to the steering correction are expected to ensure accuracy without degrading the long-term (several months) stability of TAI, and these changes are announced in advance in the BIPM Circular T.

Because TAI is computed in "real-time" every month and has operational constraints (e.g. no re-computation on a given time interval even if new data become available), it does not provide an optimal realization of TT. The BIPM therefore computes another realization TT(BIPM) in post-processing [2,3], which is based on a weighted average of the evaluations of TAI frequency by the PFS. The latest realization is TT(BIPM06) and provides the best reference which allows to estimate the stability of other time scales.

Since 1999, we have at our disposal the complete set of electronic files relative to the computation of TAI in a consistent form (see ftp://62.161.69.5/pub/tai). We use this database to study the long term stability of EAL over 96 months, from January 1999 to December 2006.

In section 2, the 1-month stability[1] of EAL is determined by two independent methods, and its long-term stability is determined with respect to TT(BIPM). In section 3, we show by comparison to TT(BIPM), existing long-term trends in the EAL frequency and discuss these trends by studying different classes of clocks participating the EAL ensemble.

II. STABILITY OF EAL AND TAI

We here consider two methods to estimate the stability of EAL and TAI. In the first approach, we compare them to TT(BIPM) which allows to obtain stability estimates for long averaging times (months up to several years). In the second approach, we use the statistical properties of the ensemble average to estimate the 1-month stability of EAL from that of the clocks participating to the ensemble.

A. Stability of TAI and EAL vs. TT(BIPM06)

TT(BIPM06) has been computed in January 2007, based on all primary frequency standards evaluations available at this date, and is available at [4]. This year's update presents two specific improvements with respect to TT(BIPM05), in addition to using one more year of data: First, the stability model of EAl, which is used to relate the PFS evaluations taken on different intervals, has been updated following results from the past study [5]: Since 2004, we consider it to be composed of a white frequency noise of $2.0\times10^{-15}/\sqrt{\tau}$, a flicker frequency noise of 0.4×10^{-15} and a random walk frequency noise of $1.0\times10^{-16}\times\sqrt{\tau}$, where τ is in days. Second, for all PFS evaluations since January 2005, the uncertainty in the link to TAI has been computed following the suggestion of the CCTF working group on PFS [6]. The latter change is the one with most effects, as it can change both the value and the uncertainty of each monthly estimation of the EAL frequency by a few parts in 10^{16}. The resulting set of monthly uncertainty values is plotted in Figure 1, and can also be considered as the accuracy level of TT(BIPM06). Note that the values of the monthly estimations of EAL frequency are shown later in Figure 6.

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[1] In this paper, stability may be used qualitatively or quantitatively. In the latter case, it means the fractional frequency instability, generally estimated by the Allan deviation $\sigma_y(\tau)$.

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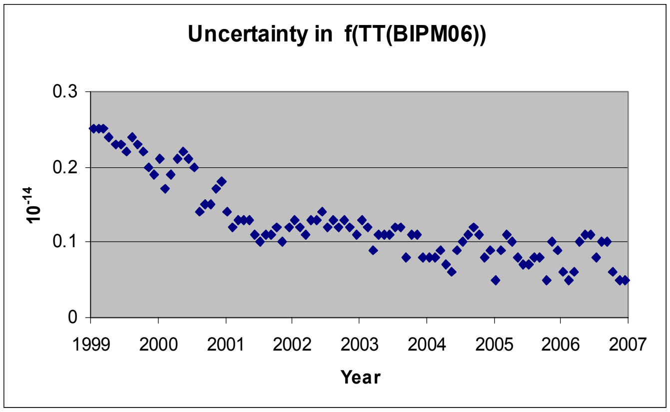

Fig. 1 Estimated accuracy of TT(BIPM06) (≡ uncertainty in EAL frequency) for each month over 1999-2006.

TT(BIPM06) may be considered as the best time reference to estimate the stability of TAI and EAL. Based on the last 14 years, we estimate the stability of TAI over the periods 1993-1999 (Figure 2A) and 2000-2006 (Figure 2B). We see very significant changes between the two periods: On the short term (1-month averaging) the improvement is significant (from around 6×10^{-16} to around 4×10^{-16}) and is linked to the improvements in EAL detailed below. For the long term (one year averaging and above), the improvement is by a factor of 3 to 4 and mostly reflects the dramatic change in PFS brought by the emergence and widespread use of Cs fountains. These have widely improved the uncertainty in evaluating EAL frequency, therefore the ability to steer EAL to obtain an accurate TAI.

Fig. 3 shows the stability of EAL vs. TT(BIPM06) over the period 2000-2006. We can see that the 1-month stability is around 4×10^{-16} but the long-term behavior is characteristic of noise processes which are redder than pure Random walk in frequency. This will be addressed in section 3.

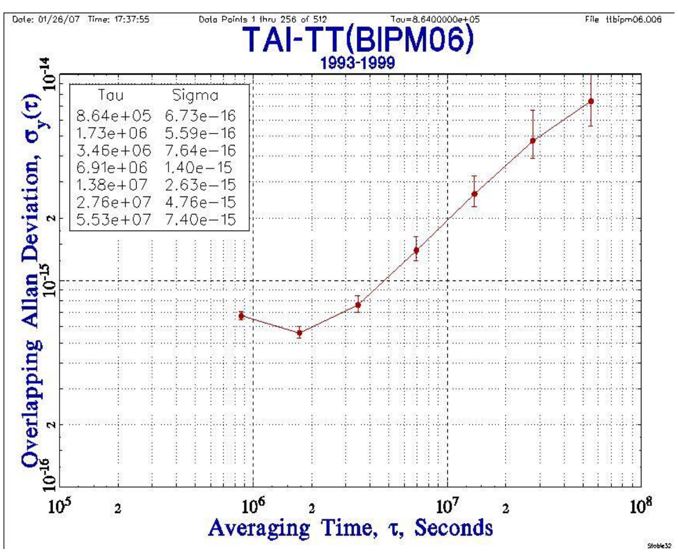

Fig. 2A: Stability of TAI vs. TT(BIPM06) over the period 1993-1999

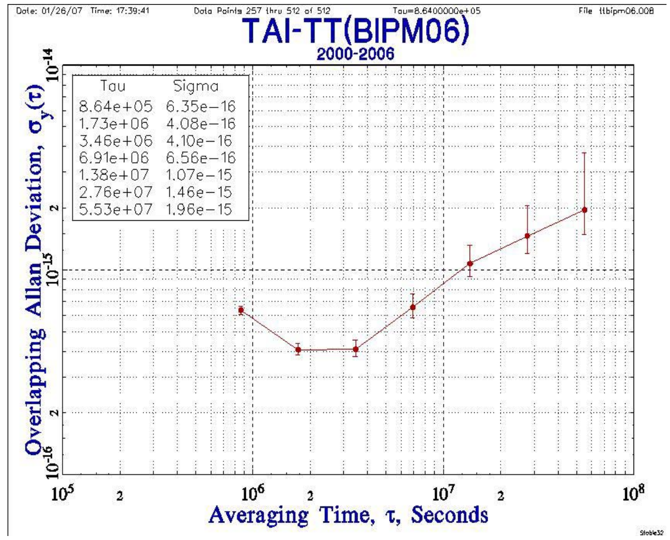

Fig. 2B: Stability of TAI vs. TT(BIPM06) over the period 2000-2006

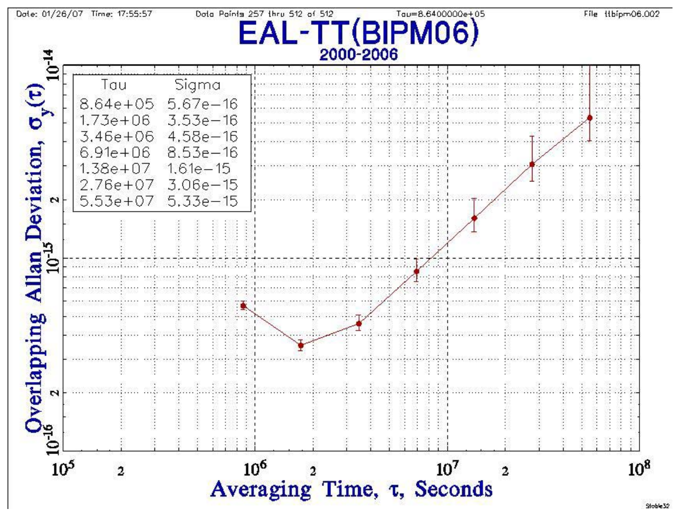

Fig. 3: Stability of EAL vs. TT(BIPM06) over the period 2000-2006

B. *Stability of EAL by the clock ensemble*

The BIPM clock database contains, for each month j, the series (w_{ij}) of weights attributed to clock i and their rates (r_{ij}) with respect to EAL over this month. From these we compute the standard deviation of the monthly rates, obtained with 11 samples (this month and the past 10), σ(11, 1month), and we may infer the Allan deviation σ_y(1month) by σ_y^2(1month) = σ^2(11, 1month) / B_1, where B_1 is a constant (e.g. about 1.9 for flicker frequency modulation) [7]. Because EAL is more stable than each individual clock, this provides a good estimate of the relative frequency stability of each clock for the month j. Then, assuming no correlation between the clocks, we may estimate the stability Σ_j of EAL over month j by $\Sigma_j^2 = \Sigma_i(w_{ij}\ \sigma_{ij}^2) / \Sigma_i(w_{ij})$. In this study, we perform this computation considering only clocks that were continuously operating for the 11-month period. In the actual EAL computation, all clocks present for at least the past 4 months are taken into consideration for obtaining some weight. The difference is small, however, because the set of 11-month clocks generally accounts for about 90% of the total weight in EAL computation (see figures 4A and 4B).

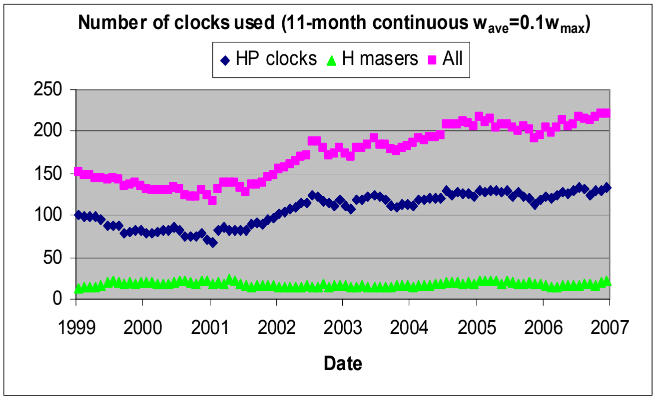

Fig. 4A: Number of "good clocks" (see text for details) participating to EAL over the period 1999-2006

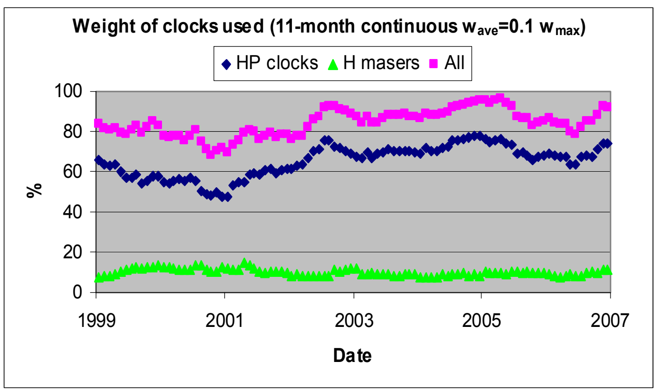

Fig. 4B: Cumulated weight of the "good clocks" (see text for details) participating to EAL over the period 1999-2006

In the period considered in this paper, there were several changes that affect EAL: Until December 2000, the maximum weight of a clock was set to a fixed value (0.7%) but from January 2001, the maximum weight was set to 2/N, where N is the number of weighted clocks, then it was set to 2.5/N from July 2002. To these discrete changes, we can add the significant and steady increase in the number of reported clocks. In addition the clocks are found to be significantly more reliable in recent years: The number of clocks continuously operating for 11 months and gaining at least one tenth of the maximum weight nearly doubled in four years, from about 120 end 2000 to about 220 end 2004 (see Figure 4A).

These changes are reflected in the stability estimation Σ_j of EAL, shown in Figure 5: The impact of the 2001 change in the weighting scheme is very visible, as is the impact of the large increase in the number of clocks between 2001 and 2003. Similarly, though much less visible, the impact of the change in the weighting scheme in July 2002, as well as of the small variations in the number of good clocks since 2003, can be seen on Figure 5.

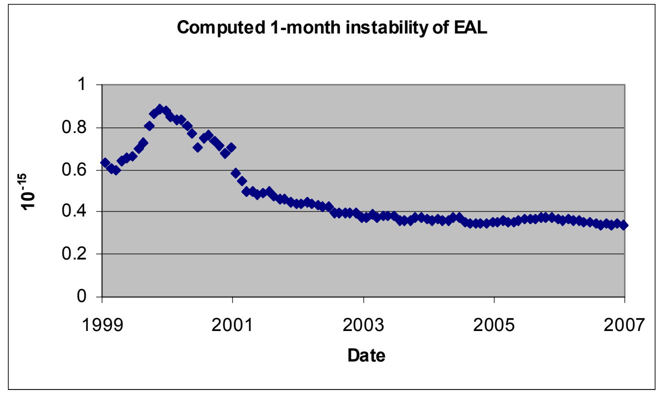

Fig. 5: 1-month stability of EAL, Σ_j, estimated from the clock ensemble over the period 1999-2006

III. LONG-TERM INSTABILITIES OF EAL

It has been noticed for years that some systematic long-term trends seem to occur in EAL. A reliable estimation of the true behavior of EAL is with respect to TT(BIPM), and is represented in Figure 6 for TT(BIPM06). We see that, although the general tendency is to a decrease of the rate of EAL vs TT(BIPM), some significant variations do occur e.g. from early-1997 to mid-1998, and also, though less clearly, in early 2004 or, may be, end 2006.

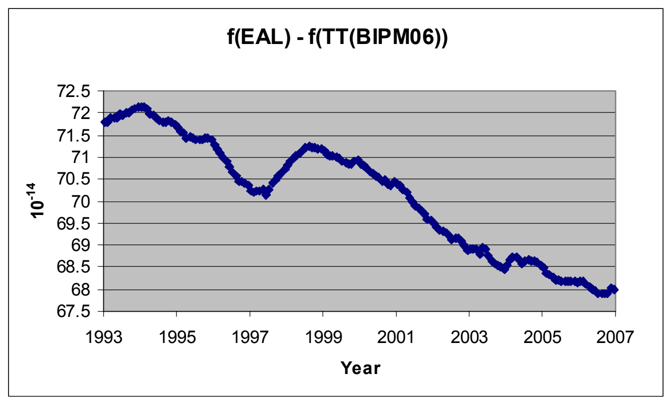

Fig. 6: Rate of EAL with respect to TT(BIPM06) over the period 1993-2006

Because the rate of EAL is that of a weighted ensemble of clocks, we investigate whether groups of clocks may display some systematic behavior. As a first approach, we distinguish the two main groups: The Cs clocks Symmetricom 5071A (earlier known as HP-Agilent, denoted HP in the following) and the hydrogen masers (denoted HM in the following). For each group, we compute the average rate drift with respect to TT(BIPM). This is computed each month using the past 11 months, i.e. the drift value for month m is the mean value of the drift over 11 months ending with m. For each computation, we keep only clocks which had a continuous operation over the 11 months and whose average weight was at least 10% of the maximum weight. The results are presented in Figures 7A and 7B.

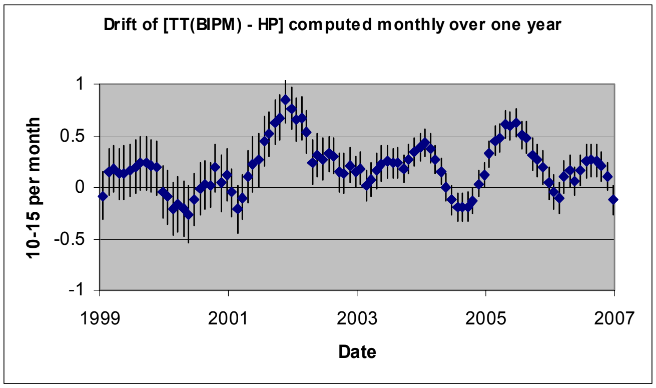

Fig. 7A: Average drift vs. TT(BIPM) for one clock in the HP set, computed monthly over the past year of continuous operation

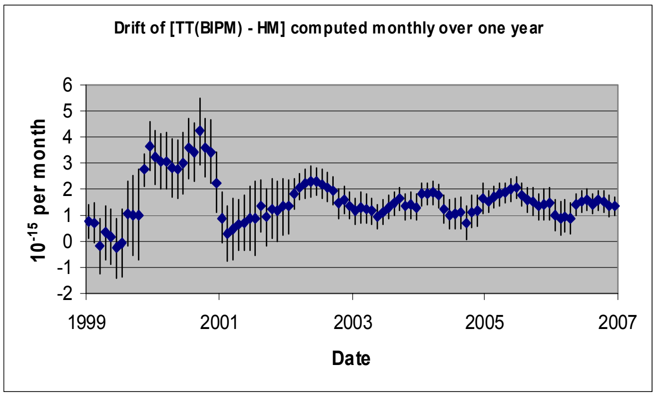

Fig. 7B: Average drift vs. TT(BIPM) for one clock in the HM set, computed monthly over the past year of continuous operation

We see (Figure 7B) that the average drift for HM seems to be both quite large (of order $1\text{x}10^{-15}$ per month) and significantly biased, having the same sign for nearly the whole period. On the other hand, the average drift for HP (Figure 7A) seems to be both smaller (of order a few 10^{-16} per month) and less biased, its sign changing occasionally over the period. However HM make 10% of EAL while HP make typically 70% of EAL (Figure 4B). Thus the net result on EAL (Figure 8) of the observed drifts in HM and HP is represented by an average drift with a significant bias (about $2\text{-}3\text{x}10^{-16}$ per month) of which only a part originates from the masers. The drift also varies significantly on time scales of years, with peak-to-peak variations over the whole period close to $1\text{x}10^{-15}$ per month.

Such long-term trends correspond to a level of instability in EAL of about $2\text{x}10^{-15}$ for 6-month averaging time (see Figure 3). This is very significant when comparing different evaluations of a Cs fountain at different periods, or when using different PFS evaluations to estimate TAI accuracy. Although there may be a tendency to smaller systematic effects over the very last year, the effects are now comparatively more important when we consider that the short-term instability of EAL improves over time, and that the accuracy of the PFS also improves. In the worst case, these effects might be large enough to offset 1-month stability improvements e.g. from a larger number of clocks. If such systematic effects can be associated with a physical mechanism, they should be taken into account in the frequency prediction of the EAL algorithm.

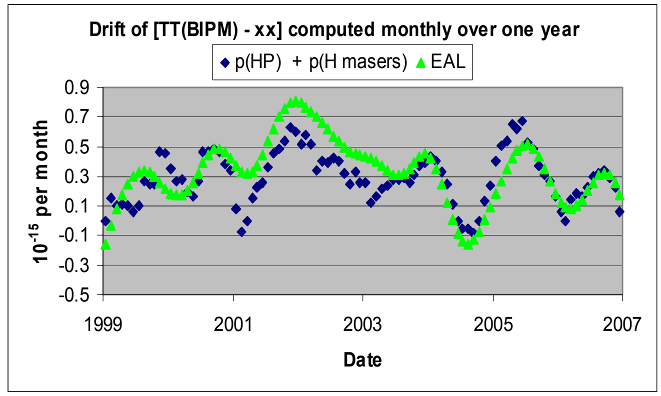

Fig. 8: Varying long-term drifts in EAL compared to the weighted combination of the drifts of the main two clock ensembles.

IV. CONCLUSION

By comparison to TT(BIPM06), we estimate the instability of EAL and TAI over the long term (one month to two years) and evidence the improvements over past years. By a reanalysis of all clock data used in the computation of TAI over 1999-2006, we have also estimated the 1-month instability of EAL to be at or below $4\text{x}10^{-16}$ since 2003 and that of TAI is similar

Some long-term trends are evident when comparing EAL to TT(BIPM). A background frequency drift of about $1\text{x}10^{-16}$/ month or slightly more may be due to H-masers, and random fluctuations with characteristic times of years may be due to the ensemble of Cs clocks. These trends correspond to a level of instability of about $2\text{x}10^{-15}$ for 6-month averaging time and may become a limiting factor to the improvement of EAL and TAI.

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